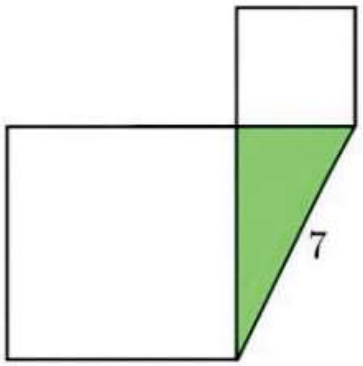
Student Name: _____

Please write your name on *every* page.

6 Section F

F1

Two squares are arranged as shown in the diagram. The sum of the side lengths of the two squares is 9 and the hypotenuse of the right triangle is 7. Find the area of the shaded region.



Answer to F1: _____

F2

Five points lie within a square of side length 1, including possibly along the edges. What is the largest possible distance between the two closest points?

Answer to F2: _____

F3

If $\frac{1}{x+2026} = 2026$, then what is $\frac{1}{x+2027}$?

Answer to F3: _____

F4

For how many positive integers $k < 2026$ is the expression $x^3 - k^2x$ always divisible by 3 for any integer x?

Answer to F4: _____

Student Name: _____

Please write your name on *every* page.

F5

A sequence is constructed as follows: starting with 0, take what you have so far, say a, and create a' which has a 1 whenever a has a 0 and has a 0 whenever a has a 1. The next piece of your construction will be a followed by a'. For example, the first few pieces of the construction are $0, 01, 0110$, and 01101001. The last piece represents the first eight terms of the sequence. In the first 2026 terms of the sequence, how many more 1s are there than 0s?

Answer to F5: _____

F6

How many pairs of integers (x, y) satisfy $1/x + 1/y = 1/6$?

Answer to F6: _____

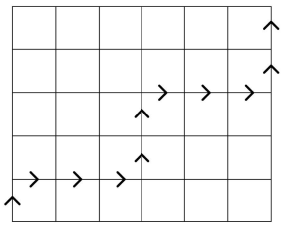
F7

Find $\frac{2\times3}{1\times4} + \frac{5\times6}{4\times7} + \frac{8\times9}{7\times10} + \cdots + \frac{98\times99}{97\times100}$

Answer to F7: _____

F8

The following is a path from the bottom left to top right corner of a 5×6 grid consisting only of movements upward and rightward. The path has 4 turns.



Find the largest value of $m+n$ with $1 \leq m, n \leq 2026$ such that in an $m \times n$ grid, the probability of a random path from bottom left to top right consisting of only up and right movements having an odd number of turns is exactly $\frac{1}{2}$.

Answer to F8: _____