

Student Name: _____

Please write your name on *every* page.

6 Section F

F1

At a movie theater, buying 13 popcorns and 16 drinks costs \$184. Buying 17 popcorns and 11 drinks costs 7 dollars more. How much would a popcorn and drink cost you in total?

Answer to F1: _____

F2

On a standard six-sided board game die, opposite sides add up to 7. When Rachelle rolls such a die, she is only able to see the top and front faces of the die. If she rolls two such dice, what is the probability that all four faces she sees are different?

Answer to F2: _____

F3

Find the units digit of $1 + 2 + 3 + \dots + 2025$.

Answer to F3: _____

F4

Abby and Betty are playing a game together on the chalkboard. On the first round, Abby draws the number 1 on the chalkboard, and then Betty can draw the number 2 at any position in the chalkboard to form a new two digit number (i.e. 12 or 21). On each subsequent round, Abby continues to add a digit of 1 to the number in any position (for example from 12 she could make 112 or 121), and then Betty adds a digit of 2 to the number. After playing x rounds, Abby wins if the number is divisible by a perfect square greater than 1, and Betty wins otherwise. What is the smallest value of x such that Abby will always win?

Answer to F4: _____

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F5

Consider a semicircle with base AB and center O , with radius 2. Bisect OA at C and let D be the intersection of the line through C perpendicular to AB with the arc of the semicircle. Construct EF on the other side in the same way. Fold region ACD across the line CD and region BEF across EF to produce arcs OD and OF . What is the area of the curved triangle ODF ?

Answer to F5: _____

F6

Two random positive integers a and b are chosen from between 1 and 10 inclusive. What is the percentage chance that the line segment in the cartesian plane from $(0, 0)$ to (a, b) contains a third point with integer coordinates?

Answer to F6: _____

F7

A gumball machine contains 100 gumballs, including exactly one rare rainbow gumball. Putting a quarter into the gumball machine dispenses a random gumball. If it is not the rainbow gumball, an additional rainbow gumball will be added to the machine to improve the odds, and you get to keep the gumball you were given. Jackson repeatedly puts quarters into the gumball machine until it gives him a rainbow gumball, and in total he ends up spending \$20.25. When expressed in lowest terms, the denominator of the probability of this occurring has n zeroes at the end. Compute the value of n .

Answer to F7: _____

F8

Let p and q be the roots of the quadratic equation $x^2 + bx + c = 0$. Find the smallest pair of positive integers (b, c) for which the expression

$$p^4 - p^3q + p^2q^2 - pq^3 + q^4$$

is a multiple of 2025.

Answer to F8: _____