Student Name:_____

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Section D

D1
How many factors are there for 2024?

Solution:
Recognize that the prime factorization of 2024 = 2^3 x 11 x 23. Thus the total number of factors of 2024 is 4 x 2 x 2 = 16.

Answer to D1: 16

D2
How many different ways are there to arrange the letters in "ILOVEMATH"?

Solution:
Note that since each letter is distinct, this is a simple permutation problem, and thus there are 9! = 362880 ways to arrange the letters

Answer to D2: 362880

D3.
Two fair dice are rolled. What is the probability that the sum of the two face up sides is greater or equal to the product of the two face up sides.

Solution:
Consider that the only scenario where the sum is greater or equal to the product is when at least one 1 is rolled or when two 2's are rolled. There are 11 combinations involving at least 1 one(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(3,1)(4,1)(5,1),(6,1), and there is one combination of two 2's (2, 2). There are 6 x 6 = 36 total combinations of two dice. Thus, there is a 12/26 = ⅓ probability.

Answer to D3: 1/3

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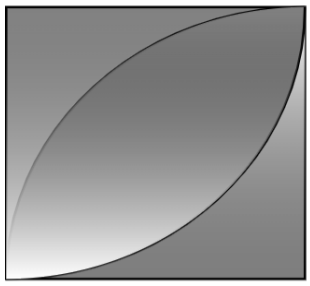
D4

In the square below with side length of 2, two quarter circles are drawn at opposite corners of the square, and that they have radius equal to the side length of the square



Find the area of the intersection between the two quarter circles (the middle area)

Solution:

Recognize that the top left area and the bottom right area are identical in size. Note that the top left area is the difference between the area of the square and the quarter circle. The area of the square is 2 x 2 = 4, and the area of the quarter circle = $\frac{1}{4}\pi 2^2 = \pi$ .Thus the area of the two outer areas is $2(4 - \pi) = 8 - 2\pi$, and thus the area of the intersection is $4 - (8 - 2\pi) = 2\pi - 4$

Answer to D4: $2\pi - 4$

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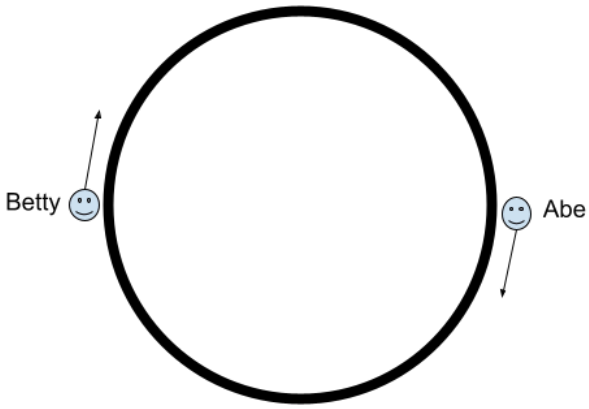
D5

On a circular track, Abe and Betty start at opposite sides of the circle facing opposite directions.



Betty decides to bring a moped onto the track, and thus she travels twice as fast as Abe. If Betty completes two full laps, find the number of times she would have passed Abe.

Solution: In two laps time, Abe would have completed one lap. However, since Abe had a half lap head start, he would have completed 1.5 laps relative to Betty’s starting position. Thus Betty should have passed Abe once.

Answer to D5: 1

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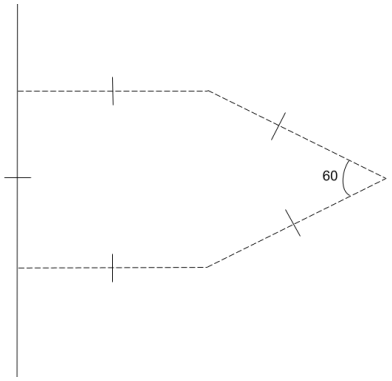
D6

Farmer Bobby is creating a pen that will attach to the side of his barn to house his pigs. He wants a design similar to the one shown below.



The dotted lines represent where new fencing should be. If Bobby has 50m of fence, what is the maximum area that Bobby can enclose within his fence.

Solution:

Consider that since the side lengths of the fences should be the same, Bobby will have to divide 50m/4 sides of the fence = 12.5m/side of the fence. Thus, the area is the area of a square with side length 12.5 and equilateral triangle with side length 12.5. Therefore, the area comes out to $12.5^2 + \frac{\sqrt{3}}{4} 12.5^2 = 156.25 + 156.25 \frac{\sqrt{3}}{4}$

Answer to D6: $156.25 + 156.25 \frac{\sqrt{3}}{4}$

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D7

An ant starts at 0 on a number line. Each day starting on day 1, the ant moves 2^n spaces to either the left or right, where n is the number of the day. What is the difference between the farthest and shortest distances the ant could be from 0 after a week?

Solution:

Note the furthest distance an ant could move is quite obviously just moving in one direction, which would equal $2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$, or in this case, 2^8 - 2 = 254. Consider that the shortest distance the ant could possibly move is 2. We can note that the ant can't possibly move a distance of 0, this would imply $2 + (2^{a1} + .. 2^{an}) = (2^{b1} + \ldots 2^{bm})$, where $a1, a2, \ldots b1, b2, \ldots. \geq 2$. However, this implies the left side is divisible by 4, while the right side isn't which clearly is a contradiction. Note that we can obtain 2 by simply travelling in one direction for the first 6 moves, and then travelling in the opposite direction on the last day, landing 2 away from the origin. Thus the farthest and shortest distances differ by 254 - 2 = 252

Answer to D7: 252

D8

Find the closest integer to the infinite expression:

$$\frac{2024}{2024 + \frac{2024}{2024 + \frac{2024}{2024 + \ldots}}}$$

Note that $x = \frac{2024}{2024 + \frac{2024}{2024 + \frac{2024}{2024 + \ldots}}} = \frac{2024}{x + 2024}$. Rearranging gets $x^2 + 2024x - 2024 = 0$, and using quadratic formula, we get x = $26\sqrt{1518}$ - 1012 ≈ 0.9995 which is closest to 1.

Answer to D8: 1