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### 3 Section C

#### C1

What is the perimeter of an equilateral triangle with side length 4?

**Solution.** An equilateral triangle has all side lengths equal, and has 3 sides, so the perimeter is  $3 \times 4 = 12$ .

Answer to C1: 12

#### C2

What is the value of  $3^4 - 4^3$ ?

**Solution.** We have  $3^4 = 9^2 = 81$  and  $4^3 = 16(4) = 64$ . Finally,  $81 - 64 = 17$ .

Answer to C2: 17

#### C3

An Egyptian fraction is a fraction written as a sum of fractions of the form  $\frac{1}{n}$ . For example,  $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ . Which regular fraction does the Egyptian fraction  $\frac{1}{4} + \frac{1}{10} + \frac{1}{15}$  correspond to?

**Solution.** We have  $\frac{1}{4} + \frac{1}{10} + \frac{1}{15} = \frac{15}{60} + \frac{6}{60} + \frac{4}{60} = \frac{25}{60} = \frac{5}{12}$ .

Answer to C3: 5/12

#### C4

A frog wants to cross a 100 metre long pond. He can swim across the pond at 5 metres per minute or he can jump across lilly pads at 8 metres per minute. Jumping is tiring, so the frog must rest for 1 minute after every 3 minutes of jumping before he is ready to jump again. How much longer does it take to swim across the pond than to jump across it?

**Solution.** It takes the frog  $100/5 = 20$  minutes to swim across the pond. If he jumps instead, it takes him  $100/8 = 12.5$  minutes of jumping, but after every 3 minutes of jumping, he must take an additional minute's rest. Rounded down,  $12.5/3 = 4$ , and so the frog rests for 4 minutes. This totals  $12.5 + 4 = 16.5$  minutes for the journey, and so the difference is  $20 - 16.5 = 3.5$  minutes.

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Answer to C4: 3.5minutes

**C5**

A sequence starts with 2, 1, 3, and each number after the second is obtained by adding the previous two numbers together. What is the 8th number in the sequence?

**Solution.** Computing the terms as  $t_n$ , we have  $t_1 = 2$ ,  $t_2 = 1$  and then

$$t_3 = t_2 + t_1 = 1 + 2 = 3$$

$$t_4 = t_3 + t_2 = 3 + 1 = 4$$

$$t_5 = t_4 + t_3 = 4 + 3 = 7$$

$$t_6 = t_5 + t_4 = 7 + 4 = 11$$

$$t_7 = t_6 + t_5 = 11 + 7 = 18$$

$$t_8 = t_7 + t_6 = 18 + 11 = 29$$

And so the 8th number in the sequence is 29.

Answer to C5: 29

**C6**

Jan is getting paid to do yard work, and her employer offers her two different payment plans for her 10 days of work. Under the first plan, Jan will make 100 dollars per day for 10 days of work, while under the second plan she will get paid 1 dollar for the first day of work and each day after that she will be paid double what she was paid on the previous day (so on day 2 she will be paid 2 dollars, on day 3 she will be paid 4 dollars, and so forth). How much more money will Jan make under the second plan than under the first?

**Solution.** Under the first plan, Jan makes  $100 \times 10 = 1000$  for her 10 days of work. Under the second plan, we may use a calculator to compute that  $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 = 2^{10} - 1 = 1023$ . Then Jan makes  $1023 - 1000 = 23$  more dollars under the second plan.

Answer to C6: 23

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**C7**

A letter of the alphabet has a value in terms of its position in the alphabet (for example, A has a value of 1 since it is first in the alphabet, B has a value of 2 since it is second, and J has a value of 10 since it is 10th). A word has a value equal to the sum of the value of its letters. For example, FROG has a value of  $6 + 18 + 15 + 7 = 46$  since F is the 6th letter of the alphabet, R is 18th, O is 15th, and G is 7th. What is the value of the word MATH?

**Solution.** Counting the letters of the alphabet, we find that M has position 13, A has 1, T has 20, and H has 8. Then MATH has value  $13 + 1 + 20 + 8 = 42$ .

Answer to C7: 42

**C8**

A machine takes in a number as an input, and outputs the number of letters in the name for that number. For example, if you input 6, the machine outputs 3 since “six” has 3 letters. If you input 3, the machine outputs 5 since “three” has 5 letters. If you take 13 and input it to the machine, feed the output back into the machine as an input, and feed the output of the second input back into the machine again, what number do you get as the third output?

**Solution.** The number 13 is written as “thirteen” and has 8 letters. The number 8 is written as “eight” and has 5 letters. Finally, the number 5 is written “five” and has 4 letters. So the third output is 4.

**Fun fact:** No matter what number you start with, if you do this procedure enough times you will always get 4. Try proving this!

Answer to C8: 4