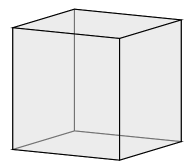
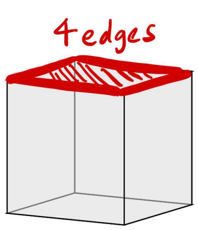
3 Section C

C1
How many edges does a cube have? You can use the picture below as a guide.

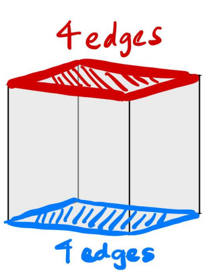


Solution. One way to solve such problems is to break the problem into smaller parts that we know how to solve.

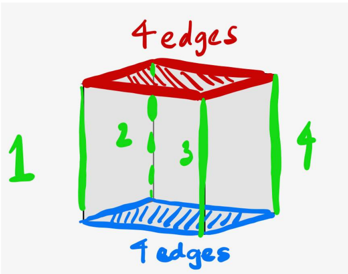
Since we know a *square* has 4 edges we can break up the problem into counting squares. First there is a square on top, which gives us 4 edges.



Next, there is a square on the bottom which gives us another 4 edges, so we have $8(=4+4)$ edges so far.



Last, we can count the remaining edges to see that there are 4 edges remaining.

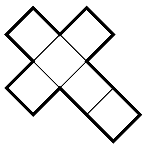


Thus, there are 12 edges in total.

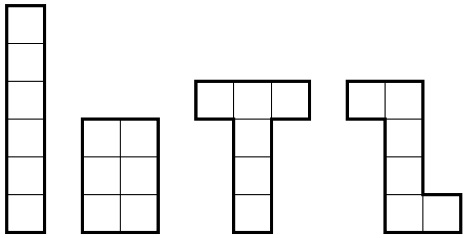
Answer to C1: 12

C2

This is an X-hexomino, which is a "net" of a cube. That means it can be folded along its edges (without making any cuts) to make a cube:



The following hexominos are called I, O, Tall T, and Tall Z, because they look like those letters. How many of these four are nets of a cube?



Solution. Try to fold the hexominos in your head. The first one (I) can never close the gaps, and the second one (O) cannot fold into anything interesting without making cuts. Therefore, only the last two can fold into a cube (check that they do work.)

Answer to C2: 2

C3
A teddy bear usually costs
)10, but today is on sale for 50% off. How much does the teddy bear cost today?

Solution. A teddy bear usually costs
(10. If today it is 50% off, then this means that today it is cheaper by 50% of
(10, which is
)5. So the cost of teddy bear today is
)$10 - \$5 = \$5$.

Answer to C3:

(5

C4
Manisha has 17 chocolates that she wants to split among herself and her 3 friends. She has two requirements:

- She wants to be fair to all her 3 friends, so they have to all get the same number of chocolates.
- She really likes chocolate, so she has to have more chocolate in the end than any of her friends.

(She cannot break a chocolate into fractional pieces, so she has to distribute a whole number of chocolates to each person.) What's the maximum number of chocolates that she can give to her 3 friends in total?

Solution. Manisha has 17 chocolates and 3 friends. She wants to give her friends the same number of chocolates but still have more chocolates than anyone else. In such problems you should look for a pattern.

For example, if Manisha gave each of her friends 1 chocolate, then she would have given $1 + 1 + 1 = 3$ chocolates away and have $17 - 3 = 14$ chocolates for herself. She has more chocolates than any of her friends here. If Manisha gave each of her friends 2 chocolates away, then she would have given $2 + 2 + 2 = 6$ chocolates away and have $17 - 6 = 11$ chocolates. She still has more than her friends here.

We notice a pattern. If Manisha gave x chocolates to each of her friends, then she gave $3 \times x$ chocolates away and has $17-3x$ chocolates remaining. If you know about inequalities, then all you have to do is solve

$$17 - 3x \geq x$$

which is the same as

$$17 \geq 4x$$

Hence the largest value of x, the number of chocolates she gives to each friend, is 4.

Otherwise, we can try cases one by one. If she gives 3 chocolates to each friend, then she has given $3 \times 3 = 9$ chocolates away and has $17 - 9 = 8$ chocolates remaining. This is okay since 8 is bigger than 3. If she gives 4 chocolates away, then she has given $4 \times 3 = 12$ chocolates away. She has $17 - 12 = 5$ chocolates remaining. So this case is also okay as 5 is bigger than 4.

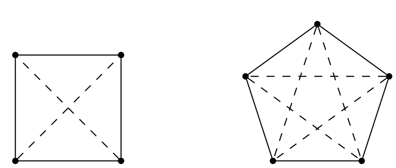
However if she gives 5 chocolates away to each friend, then she gives $5 \times 3 = 15$ chocolates away but only has $17 - 15 = 2$ chocolates left for herself. So, the largest possible number of chocolates she can give to each friends is 4.

Once we have the largest possible number of chocolates she can give to each friend, the largest possible number of chocolates she can give away is $3 \times 4 = 12$.

Answer to C4: 12

C5

A square has 2 diagonals, and a regular pentagon has 5 diagonals, as you can see from the picture below:



How many diagonals does a regular hexagon (6-sided shape) have?

Solution. One way to solve this is to draw out the hexagon and draw all its diagonals, then count the diagonals, but this can lead to a wrong answer if you aren't very careful. A more reliable method is to systematically draw the diagonals to make sure we aren't missing any. Start at a vertex and then connect it to the 3 vertices it isn't already connected to. Now look at the vertex next to it clockwise. We can also draw 3 lines to other vertices here. Moving clockwise again, we see that one of the diagonals has already been drawn, so we can draw 2 new ones. Moving again, we can now draw only 1 new diagonal, and afterwards no matter where we go we can't draw any new ones. So the hexagon has $3+3+2+1=\boxed{9}$ diagonals.

Answer to C5: 9

C6
What is the last digit of $1+2+3+\cdots+30$?

Solution. This is a big sum, so if we wanted to we count use a calculator to add each number in turn we could, but this runs the risk of making a typo and getting the wrong answer. Another way to do this is to notice that 1, 11, and 21 have the same units digit, as do 2, 12, and 22, and so forth. So we only need to find the units digit of $1+2+\cdots+10$ and then take three copies of it and add it, potentially taking the units digit of this new result. This shorter sum is easier to enter into a calculator and gives us 55 with a units digit of 5. Then 5 + 5 + 5 = 15 which has a units digit of 5, which is our answer. Alternatively, we can start taking pairs of numbers from opposite ends which sum to the same thing. 1 + 30 = 31, 2 + 29 = 31, etc. There are 15 of these pairings, so the sum will be 31(15) = 465 which can easily done by a calculator. The units digit of this is $\boxed{5}$.

Answer to C6: 5

C7

You can put exactly two $+$ symbols and exactly two $\times$ symbols in the boxes below to create an expression like $1 \times 2 + 3 \times 4 + 5$. What is the maximum possible value of this expression? (Be careful of the order of operations.)

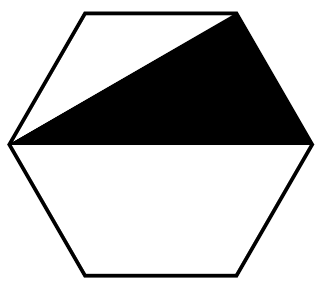
$$1\square 2\square 3\square 4\square 5$$

Solution. We could try every arrangement of + and × and choose the largest, but this would be 6 different arrangements and would be slightly annoying to calculate. A better way is to figure out when a × b > a + b. Doing some experimentation reveals that if both numbers are at least 2, multiplication will produce at least as large a result as addition. So it is most efficient to put the × symbols between bigger numbers. We get that the best order is $1 + 2 + 3 \times 4 \times 5 = \boxed{63}$ is the maximum.

Answer to C7: 63

C8

The diagram below is a regular hexagon drawn to scale. If the shaded region has area 10, then what is the area of the entire hexagon?



Solution. The easiest way to solve this question is to split the hexagon into 6 equilateral triangles about the centre by connecting pairs of opposite sides. Notice that of the 3 equilateral triangles in the top half of the hexagon, two of them are half covered by the black shaded region, and the remaining one is fully covered. This means that 4/6 parts of the top half of the hexagon are covered by the shaded region. Thus the black triangle is 2/3 of the area of half of the hexagon, or 1/3 the area of the whole hexagon. Thus the hexagon has area 3(10) = $\boxed{30}$. One could also solve this problem using angles, realising that the black triangle is a right angled triangle.

Answer to C8: 30