Student Name: ______________________________

Please write your name on *every* page.

6 Section F

F1

Find all values of x such that $1/(x-3) - 3/(x-1) + 3/(x+1) - 1/(x+3) = -3$.

Answer to F1: ____________

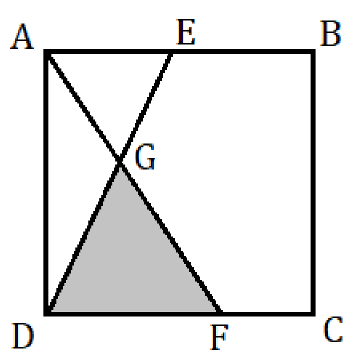
F2

The surface area of a cylinder's curved surface is 7 times larger than the surface area of one of its bases. What is the ratio of the cylinder's radius to the cylinder's height? (Write the ratio as a fraction in the form of $\frac{a}{b}$, with a and b in lowest terms.)

Answer to F2: ____________

F3

In the diagram below, ABCD is a square with side length 6. AF and DE are drawn and intersect at G such that $DF = 4$ and $AE = 3$. Find the area of the shaded triangle $\triangle DFG$.



Answer to F3: ____________

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F4

The planet Mathzorg uses a different number system, which they call the Mathzorg system, compared to Earth's base 10 number system. The number 121 in the Mathzorg system is the number 16 in base 10. What is the Mathzorg number 2020 in base 10?

Answer to F4: ____________

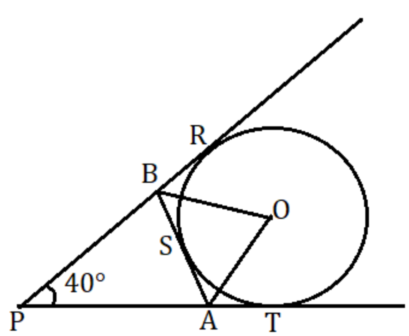
F5

Jose wants to take his water bottle with him on a hike. It is a cylinder with diameter 4 cm and height 20 cm, with an opening on the top of diameter 2 cm. The issue is that Jose's water bottle does not have a cap, and the holder on his backpack holds it sideways, so that the opening is perpendicular to the ground. With this in mind, how high can he fill his water bottle when placed right side up so that no water will spill when it is held sideways?

Answer to F5: ____________

F6

In the diagram below, O is a circle and triangle PAB is formed by three tangents to O. What is the measure of $\angle AOB$?



Answer to F6: ____________

F7

Find the greatest integer less than $\frac{1}{1^{2/3}} + \frac{1}{2^{2/3}} + \ldots + \frac{1}{1000^{2/3}}$.

Answer to F7: ____________

Student Name: ______________________________

Please write your name on *every* page.

F8

Garrett is summing some series of numbers for fun. An **arithmetic sequence** is a sequence with an initial term a and common difference d between terms, so that the sequence looks like $a, a+d, a+2d, \dots, a+(n-1)d$ after n terms.

An **arithmetic series** is the sum of an arithmetic sequence. In general, we have that

$$a + (a+d) + (a+2d) + \dots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d].$$

A **geometric sequence** is a sequence with an initial term a and common ratio r between terms, so that the sequence looks like $a, ar, ar^2, \dots, ar^{n-1}$ after n terms.

A **geometric series** is the sum of a geometric sequence. In general for $r \neq 1$, we have that

$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}.$$

Garrett is bored of these sequences, and seeks to combine them to achieve something more interesting. Starting with an initial term a (not necessarily an integer), he multiplies by 2 and then adds a common difference d (again, not necessarily an integer) to get the second term. He then takes this term and multiplies by 2 again, and then adds d again to get the third term. He continues this until he has n terms, and then he adds all of those terms together.

Upon doing this with a certain a and d, he notices a few interesting coincidences:

- $a + d$ is an integer
- the series sums to 0
- nd is a perfect number, which means that the sum of the divisors of nd (not including nd itself) equals nd. Note that a perfect number can be written in the form $2^{p-1}(2^p - 1)$ for a prime number p, but not every such product is perfect (ex: $2^{10}(2^{11} - 1) = 2096128$ is not perfect).

Find the largest $d < 100$ for which this is possible.

Answer to F8: ______________