

Student Name: _____

Please write your name on *every* page.

5 Section E

E1

What is the smallest whole number, such that, when you multiply its digits together, you get 80?

Answer to E1: _____

E2

An icosahedron is a three-dimensional solid whose faces are regular (equilateral) triangles. An icosahedron has 20 faces. How many edges does an icosahedron have?

Answer to E2: _____

E3

We define the Levenshtein Distance between two strings to be the minimum number of edits we need to turn one string into another. One edit is either:

- adding a character: CAT → CATS
- removing a character: TIME → TIE
- turning a character into another: BAKE → CAKE

As an example, the Levenshtein Distance between HELLO and MELON is 3. We may, for example, change H to M, then delete one L, and then add an N. There are other ways of achieving the same result, but the best we can do is 3 edits.

What is the Levenshtein Distance between KITTENS and TENTS?

Answer to E3: _____

E4

What is the last digit of $1 + 3 + 3^2 + 3^3 + \dots + 3^{2018}$?

Answer to E4: _____

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E5

In Sockland, there is a magical machine that sells socks. When you buy a sock from the machine, it gives you a single sock. The colour of the sock is random, and can be red or blue, with probability $1/2$ each. Bobby buys 10 socks from the machine. What is the probability that he ends up with an even number of socks of each colour?

Answer to E5: _____

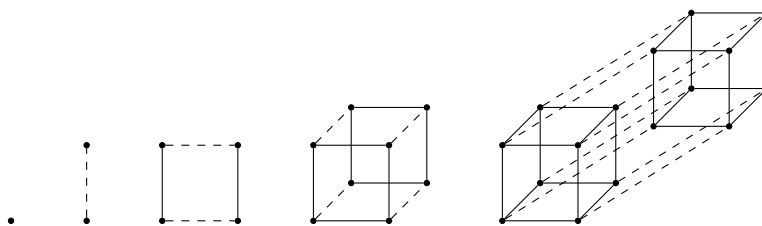
E6

Define $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. (This is the Fibonacci sequence, and it starts $1, 1, 2, 3, 5, 8, \dots$; each number is the sum of the previous two numbers.) Calculate $F_{31} \times F_{31} - F_{30} \times F_{32}$.

Answer to E6: _____

E7

A 0-hypercube is a point. A 1-hypercube is a line segment, and is formed by connecting the vertices of two 0-hypercubes. A 2-hypercube is a square, and is formed by connecting the corresponding vertices of two 1-hypercubes. Similarly, a 3-hypercube is a cube, formed by connecting the corresponding vertices of two 2-hypercubes, and a 4-hypercube is called a tesseract, formed by connecting the corresponding vertices of two 3-hypercubes. We can keep continuing this pattern. Two dimensional drawings of 0, 1, 2, 3, and 4-hypercubes are provided in this picture:



How many edges does a 6-hypercube have?

Answer to E7: _____

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E8

A **message** is a string of 0s and 1s; each digit in the message is called a **bit**. Rohan wishes to send Jasmine the 4-bit message 0101 across an unreliable communication channel. It is possible for some bits to be flipped from 0 to 1 or 1 to 0. Each bit is flipped with probability $\frac{2}{5}$.

To make it more likely the message is delivered correctly, Rohan sends each bit of the message three times, so the message he transmits is 000 111 000 111.

Jasmine will try to decipher Rohan's message by looking at each 3-bit block, and guess the original bit was:

$$\begin{cases} 0 & \text{if she sees 000, 001, 010, or 100} \\ 1 & \text{if she sees 111, 110, 101, or 011} \end{cases}$$

For instance, Jasmine could receive the message 101 111 001 101, where the underlined bits have been flipped from their original values. Jasmine would first look at the first 3 bits, and since she sees 101, would guess 1 as the bit Rohan intended to send. Jasmine continues to guess the whole message as 1101, which is not correct.

What is the probability that Jasmine will guess the correct message exactly? Write the answer in the form a^b/c^d where a and c are prime numbers and b and d are positive exponents.

Answer to E8: _____